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Control for Hexacopters: A Sliding Mode Control and PID Comparison

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Abstract

The objective of this paper is to design and to test the performance of different control strategies for vertical take-off, landing and changes in the three angles with disturbances for a hexacopter. The control strategies to be used are PID and Sliding Mode Control. The Sliding Mode Control can be implemented using a PD controller as the sliding surface. The controllers are tested through simulations.

Keywords: Robust control; sliding mode control; hexacopter; performance; PID

Control para Hexacópteros: Una Comparación de Control por Modo Deslizante y PID

Resumen

El objetivo de este artículo es diseñar y probar el funcionamiento de diferentes estrategias de control para el despegue, aterrizaje y cambios en los tres ángulos de un hexacóptero. Las estrategias de control a usarse son el PID y el Control en Modo Deslizante. El control en Modo Deslizante puede ser implementado usando un controlador PD como Superficie deslizante. Los controladores son probados mediante simulaciones.

Palabras clave: Control robusto; control por modo deslizante; hexacóptero; desempeño; PID

1. Introduction

The interest for Unmanned Aerial Vehicles (UAVs) is increasing every day. UAV´s are used in several applications such as: in military, in agricultural tasks, in industrial maintenance, and also may be employed for a wide variety of transportation operations and planning applications: monitor freeway conditions, coordination among a network of traffic signals, traveler information, emergency vehicle guidance, track vehicle movements in an intersection, and so on. [1]

UAV´s are nonlinear and present an interesting dynamics where the uncertainties on physical parameters make them an attractive and challenging control problem [2, 3]. A robust and accurate control system is essential for UAVs to successfully perform different tasks and also compensate for uncertainties and the high nonlinearities present in plane dynamics.

Sliding Mode Control (SMC) is a robust and simple procedure that allows synthesizing controllers for linear and nonlinear processes [4]. The main advantages of using sliding mode control are robustness to parameter

uncertainty, insensitivity to load disturbance and fast dynamics response. Commonly, the design of this controller depends completely on the process model, and the number of tuning parameters are in proportion to the model order. The major drawback of sliding mode control is the so called chattering phenomenon [5, 6]. Various SMC proposal have been used to control UAVs. Mohd et al [7]. M'hammed Guisser and Hicham Medromi [8]. Nader et al [9]. Benallegue et al [10].

This work shows a general and simple SMC strategy that can be applied to hexacopters. The main impact of this work is that the proposed SMC is based on easy concepts. The controller algorithm can be implemented using a classical PD controller as the sliding surface. Therefore, an existing controller structure is utilized, which no need complex calculations, and additionally it presents low computational cost to achieve the control signal. Two different control strategies are tested, in such a way that the system can take-off and land even though disturbances. The control strategies compared are PID and Sliding Mode Control, since PID represents an standard and it is the most used controller in industry. The performance and robustness analysis to tracking and regulation tasks are presented through simulations.

This paper is organized as follows. Section 2 presents the hexacopter dynamic model. Section 3 presents the fundamentals and the formulation of the Sliding Mode Control. In section 4 different simulations are developed, and their discussion are presented. Finally, section 5 contains the conclusions.

2. Hexacopter dynamic model

The scheme of the hexacopter studied here is shown in Figure 1. The equations of motion are given in [11].
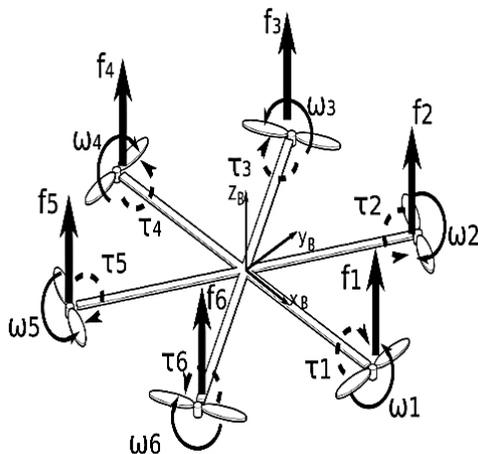

Figure 1. Scheme of the hexacopter

The hexacopter has twelve states, which are the following:

$$X = \left[x, \dot{x}, y, \dot{y}, z, \dot{z}, \emptyset, \dot{\emptyset}, \theta, \dot{\theta}, \psi, \dot{\psi}\right] \qquad (1)$$

Where, x, y and z are the position in axes. $\dot{x}$, $\dot{y}$, and $\dot{z}$ are the speed in the axes. $\emptyset$, θ and ψ are the roll, pitch, and yaw angles respectively, and the parameters are $\dot{\emptyset}$, $\dot{\theta}$ and $\dot{\psi}$ the speed for angles.

$$\ddot{x} = (\cos\emptyset \sin\theta \cos\psi + \sin\emptyset \sin\psi)\frac{1}{m}U_1 \qquad (2)$$

$$\ddot{y} = (\cos\emptyset \sin\theta \sin\psi - \sin\emptyset \cos\psi)\frac{1}{m}U_1 \qquad (3)$$

$$\ddot{z} = -g + (\cos\emptyset \cos\theta)\frac{1}{m}U_1 \qquad (4)$$

$$\ddot{\emptyset} = \dot{\theta}\dot{\psi}\left(\frac{I_y - I_z}{I_x}\right) - \frac{J_r}{I_x}\dot{\theta}\omega + \frac{l}{I_x}U_2 \qquad (5)$$

$$\ddot{\theta} = \dot{\emptyset}\dot{\psi}\left(\frac{I_z - I_x}{I_y}\right) - \frac{J_r}{I_y}\dot{\emptyset}\omega\frac{l}{I_y}U_3 \qquad (6)$$

$$\ddot{\psi} = \dot{\emptyset}\dot{\theta}\left(\frac{I_x - I_y}{I_z}\right) + \frac{l}{I_z}U_4 \qquad (7)$$

The input signal U_1 is the total drag of the rotors. U_2, U_3 and U_4 are the moments for pitch, roll and yaw respectively. m represents the mass of the hexacopter, J_r is the inertia of the rotor and Ix, Iy and Iz are the inertia of the quad rotor in 'x', 'y' and 'z' respectively.

The input signals are described from the equation (8) to (11).

$$U_1 = b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2 + \omega_5^2 + \omega_6^2) \qquad (8)$$

$$U_2 = \frac{\sqrt{3}}{2}(\omega_2^2 + \omega_3^2 - \omega_5^2 - \omega_6^2) \qquad (9)$$

$$U_3 = \frac{b}{2}(2\omega_1^2 + \omega_2^2 - \omega_3^2 - 2\omega_4^2 - \omega_5^2 + \omega_6^2) \qquad (10)$$

$$U_4 = b(-\omega_1^2 + \omega_2^2 - \omega_3^2 + \omega_4^2 - \omega_5^2 + \omega_6^2) \qquad (11)$$

Where $\omega_1, \omega_2, \omega_3, \omega_4, \omega_5$ and ω_6 are the angular speed for each rotor, b is the drag factor and l is the length of the center of the hexacopter to the rotor.

All parameters within equations (2) to (11) are declared in [10].

From equations (8) to (11) the factor ω can be obtained as follows:

$$\begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \\ \omega_5^2 \\ \omega_6^2 \end{bmatrix} = (M)^+ \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

Where M is

$$M = \begin{bmatrix} b & b & b & b & b & b \\ 0 & \dfrac{\sqrt{3}}{2}b & \dfrac{\sqrt{3}}{2}b & 0 & -\dfrac{\sqrt{3}}{2}b & -\dfrac{\sqrt{3}}{2}b \\ b & \dfrac{b}{2} & -\dfrac{b}{2} & -b & -\dfrac{b}{2} & \dfrac{b}{2} \\ -b & b & -b & b & -b & b \end{bmatrix}$$

And $(M)^+$ is the pseudoinverse of the matrix M.

Finally the value of ω is:

$$\omega = -\omega_1 + \omega_2 - \omega_3 + \omega_4 - \omega_5 + \omega_6 \quad (12)$$

3. Sliding Mode Controller Design

The system is going to be controlled using four control equations. The signal U_1 is going to be used to control the altitude while the signals U_2, U_3 and U_4 are the controllers for roll, pitch and yaw angles of the system. The main idea to control the altitude by using sliding modes control concepts [4].

Firstly, let us use [5] the following sliding surface:

$$s = \dot{e} + \lambda e \quad (13)$$

Where λ is a tuning parameter.

The error equation for the altitude is given by:

$$e = z_d - z \quad (14)$$

Where z_d is the desired state and z is the measured state. By replacing (14) in (13) the following result is obtained

$$s = (\dot{z}_d - \dot{z}) + \lambda(z_d - z) \quad (15)$$

The time derivative of the sliding surface is:

$$\dot{s} = \ddot{e} + \lambda \dot{e} \quad (16)$$

And replacing Eq. (14) into Eq. (16), it is obtained:

$$\dot{s} = (\ddot{z}_d - \ddot{z}) + \lambda(\dot{z}_d - \dot{z}) \quad (17)$$

Substituting Eq.(4) into Eq.(17)

$$\dot{s} = \left(\ddot{z}_d + g - (\cos\varnothing\cos\psi)\frac{1}{m}U_1\right) + \lambda(\dot{z}_d - \dot{z}) \quad (18)$$

Considering $U_1 = u_{eq}$ since the systems is in sliding condition, the above equation becomes:

$$\dot{s} = \left(\ddot{z}_d + g - (\cos\varnothing\cos\psi)\frac{1}{m}U_1\right) + \lambda(\dot{z}_d - \dot{z}) \quad (19)$$

As ṡ=0; then u_{eq} becomes:

$$u_{eq} = [\ddot{z}_d + g + \lambda(\dot{z}_d - \dot{z})]\frac{m}{\cos\varnothing\cos\psi} \quad (20)$$

The complete controller can be written as follows:

$$U_1 = [\ddot{z}_d + g + \lambda(\dot{z}_d - \dot{z})]\frac{m}{\cos\varnothing\cos\psi} + k_D sign(s) \quad (21)$$

To design u_D, a Lyapunov function V is defined. This function must be positive-definite.

$$V = \frac{1}{2}s^2 > 0 \quad (22)$$

The time derivative of the function V must be negative-definite.

$$\dot{V} = s\dot{s} < 0 \quad (23)$$

The reaching condition is given if the inequality $\dot{V} < 0$, it is satisfied if:

$$k_D > 0 \ \forall \ t > 0 \quad (24)$$

And u_D is:

$$u_D = k_D sign(s) \quad (25)$$

To reduce the chattering problem, u_D can be rewritten as follows

$$u_D = k_D \frac{s}{|s| + \delta} \quad (26)$$

Thus, the complete controller equation is

$$U_1 = [\ddot{z}_d + g + \lambda(\dot{z}_d - \dot{z})]\frac{m}{\cos\varnothing\cos\psi} + k_D\frac{s}{|s| + \delta} \quad (27)$$

The same procedure is followed to get the controllers for roll, pitch and yaw.

$$u_{eq\emptyset} = \left[\ddot{\emptyset}_d + \frac{J_r}{I_x}\dot{\theta}\omega - \dot{\theta}\dot{\psi}\left(\frac{I_y - I_z}{I_x}\right) + \lambda(\dot{\emptyset}_d - \dot{\emptyset})\right]\frac{I_x}{l}$$

$$(28)$$

$$u_{eq\theta} = \left[\ddot{\theta}_d + \frac{J_r}{I_y}\dot{\emptyset}\omega - \dot{\emptyset}\dot{\psi}\left(\frac{I_z - I_x}{I_y}\right) + \lambda(\dot{\theta}_d - \dot{\theta})\right]\frac{I_y}{l}$$

$$(29)$$

$$u_{eq\psi} = \left[\ddot{\psi}_d - \dot{\emptyset}\dot{\theta}\left(\frac{I_x - I_y}{I_z}\right) + \lambda(\dot{\psi}_d - \dot{\psi})\right]I_z$$

$$(30)$$

The discontinuous part of these controllers (Figure 2) are calculated in a similar way as in the altitude controller case, and the equations are analogous to equation (26), considering that the sliding surface depends on the error for each case.

Figure 2. Implementation of the proposed approach using an industrial PD controller
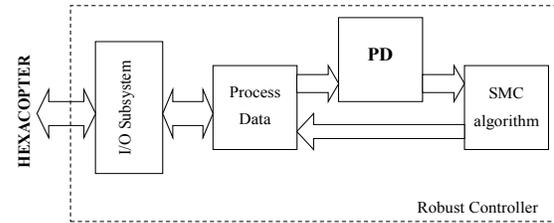
4. Simulation results

In this section, the proposed approach and the PID controller are tested by simulations, firstly a performance comparison of the SMC against the PID for tracking and regulation tasks is revised, and secondly robustness testing are done: for mass changes, for modelling errors, and finally is considered the effect of noise. The hexacopter nominal parameters are specified in table 1.

Table 1. Hexacopter nominal parameters

Name	Description	Value	Unity
m	Mass	1.83	Kg
b	Drag factor	2.98e-6	Ns^2
l	Length of the center of mass to the rotor	0.30	m
g	Gravity	9.8	m/s^2
Jr	Rotational Inertia	3.357e-5	kgm^2
Ix, Iy	x, y inertia	0.0216	kgm^2
Iz	z inertia	0.0432	kgm^2

The PID controller was tuned using Simulink tools. For the SMC the tunings were obtained by trial and error, the tunings parameters for both controllers are in tables 2 and 3.

Table 2. SMC tuning parameters

Parameter	z	$\emptyset$ (Roll)	θ (Pitch)	ψ (Yaw)
λ	1.5	30	30	5
k_D	60	30	30	5
Δ	0.25	0.2	0.2	0.2

Table 3. PID tuning parameters

Parameter	z	$\emptyset$ (Roll)	θ (Pitch)	ψ (Yaw)
Kp	50	50	2.35	50
Ki	0.02	0.02	0.35	0.02
Kd	5	5	1.45	5
Filter	150	150	39.86	100

4.1 Comparison SMC vs PID

In order to make a quantitative measure of the performance of both controllers the IAE Index is used:

$$IAE = \int |e| dt$$

(31)

Δ% is defined by the following equation:

$$\Delta\% = \left| \frac{IAE1 - IAE2}{\left(\frac{IAE1 + IAE2}{2}\right)} \right| * 100$$

(32)

The comparison is done in such a way that SMC is compared with the best PID. Tables 4 and 5 show the results with and without disturbances.

Table 4. Comparison SMC vs PID without disturbances

	SMC	PID	Δ%
IAE z	3.95	4.26	7.62
IAE ϕ	1.54e-04	8.19e-05	61.14
IAE θ	1.54e-04	8.19e-05	61.12
IAE ψ	7.89e-04	1.6e-04	181.25

Table 5. Comparison SMC vs PID with disturbances

	SMC	PID	Δ %
IAE z	4.18	4.26	1.89
IAE ϕ	1.59e-04	8.51e-04	60.75
IAE θ	1.59e-04	3.04e-02	61.93
IAE ψ	8.20e-04	1.6e-04	181.26

Figures 3 represent the system responses for each controller when external disturbances are considered. The SMC presented a better performance than the PID controller for all cases, it reached the desired reference in shorter time and also zero steady-state error, both tracking and regulation tasks are accomplished. For the case of altitude, the PID controller fails to achieve both tasks.
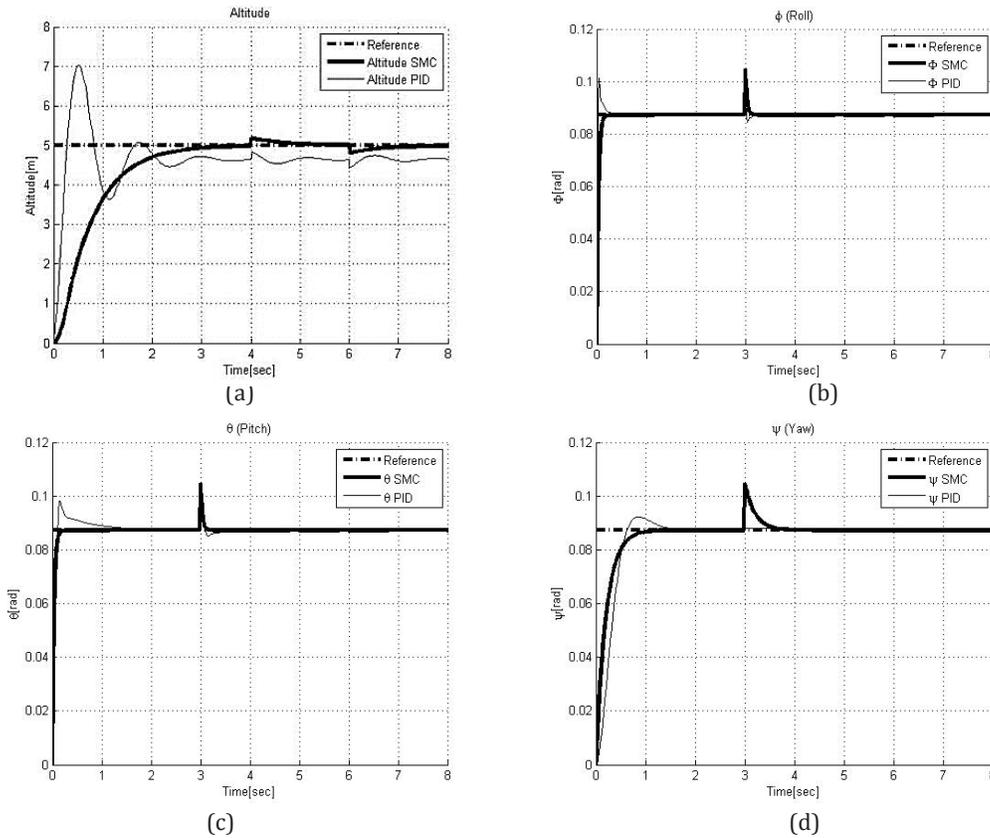

Figure 3. (a) Altitude response with disturbances (b) ϕ response with disturbances (c) θ response with disturbances (d) ψ response with disturbances

4.2 Robustness testing

One of the main advantages of the SMC is the robustness to modeling errors and uncertainties. In this part, modeling errors in parameters affecting considered are mass and inertia moments in 'x', 'y' and 'z' axes.

4.2.1 Mass changes

The mass is a parameter that directly influences the system and specifically its stability. The mass is modified from its nominal value up to 10 kg. Figures 4 (a) and 6 (b) show how the mass changes can influence the Hexacopter response for each controller.

(a)   (b)
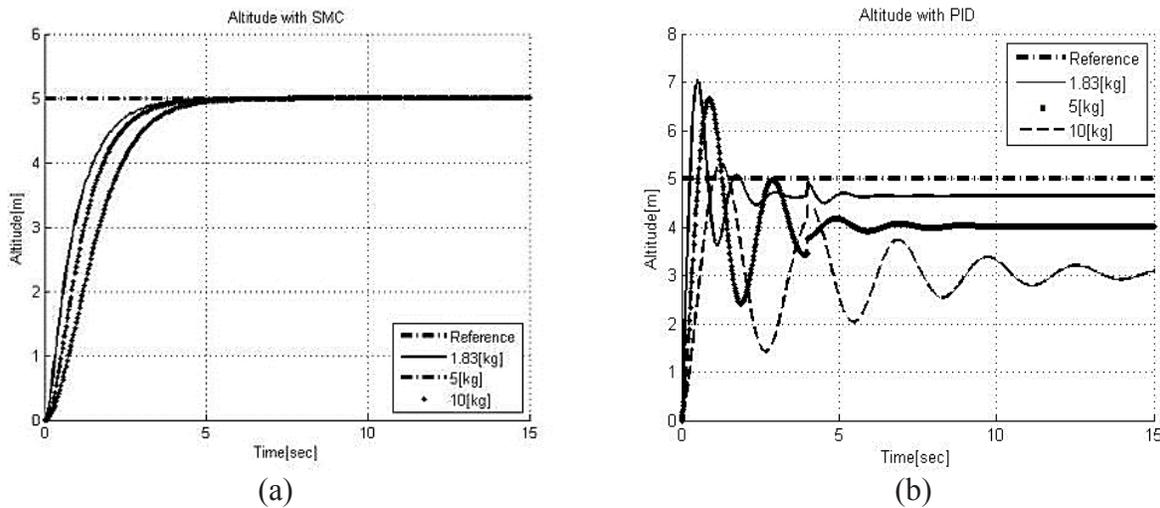
Figure 4. (a) SMC. (b) PID

As it is shown in the previous figures, the mass changes for SMC case only affect settling time. In the PID case is seen that when mass increases, the steady-state errors and oscillations are growing, and they can cause instability.

Figures 6 illustrates 3D trajectories and controller responses for the SMC and PID control when variations in the mass are made up to 6 times the nominal value, as already explained above. SMC has a smooth response throughout the trajectory and reaches the desired reference value for all variations, the controller responses for each variation are also soft and are in a range of values that can be manageable and implemented. Meanwhile, the PID controller presents oscillatory responses, with steady-state error. As shown in figure 5, the initial value PID control signal is too high which means that the force or thrust required to take-off is at least 20 times greater than that required in SMC.

(a)   (b)

(c)   (d)
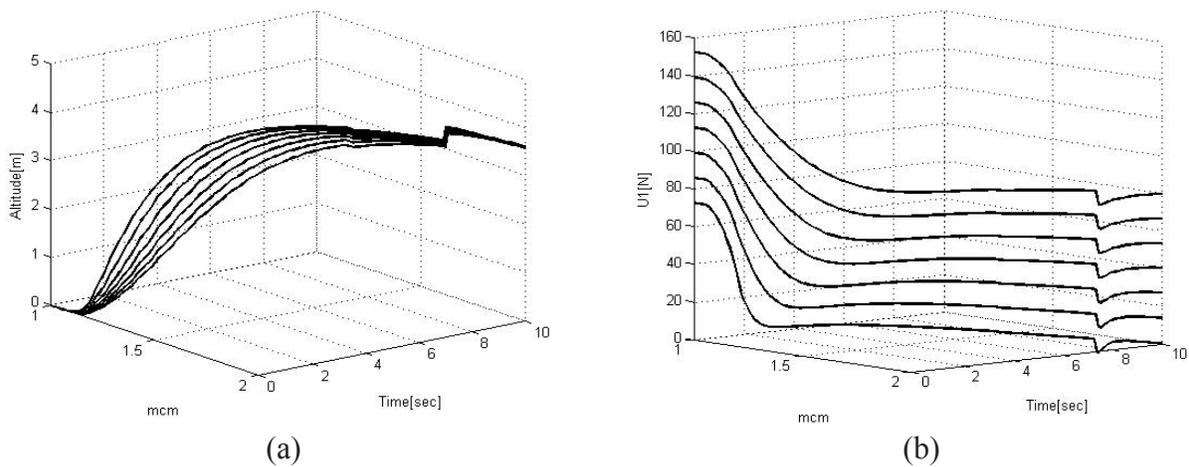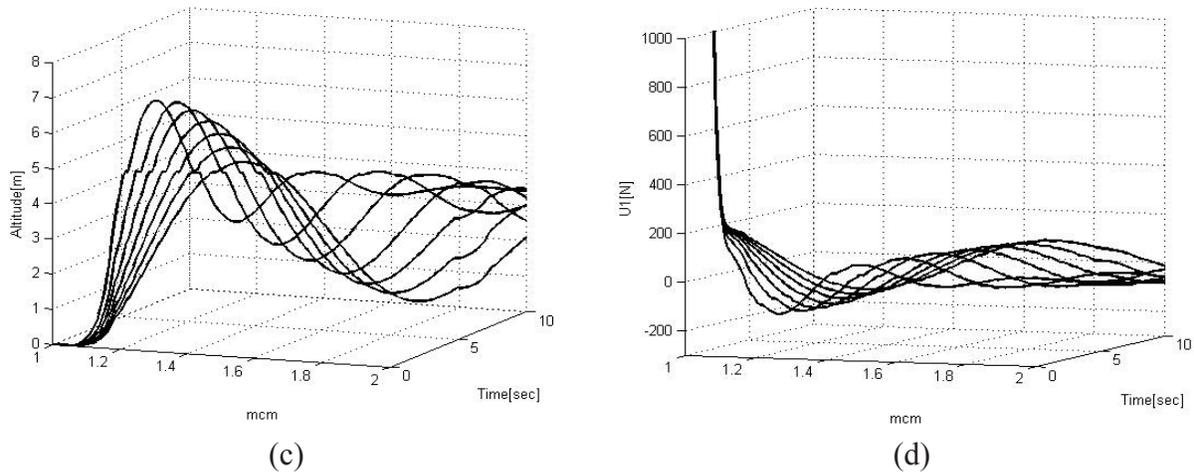
Figure 5. Altitude trajectories and Controller responses for different values
of mass. (a) and (b) for SMC. (c) and (d) for PID.

4.2.2 Modeling errors

In this section errors in the inertia moments of the different axes, are considered. These values are modified around 50% of the original values, the responses have depicted in figure 5. Figure 6 shows that the SMC reaches the desired reference value despite the changes that occurred in inertia moments, the settling time increased, but the results are still satisfactory. Contrasting, the PID controller has oscillations and it has a steady-state error.
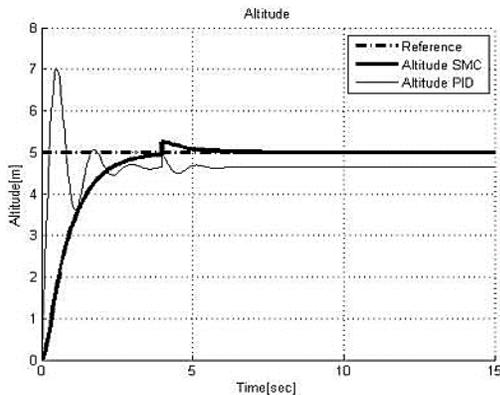
Figure 6. Modeling errors and disturbances

Taking into account all the features and advantages that have been presented for the SMC, the next experiment includes changes in angles and altitude. The nominal values are z = 5m, Roll=5° (π/36 rad), Pitch=5° (π/36 rad), Yaw=5° (π /36) and disturbances of 30% for altitude and π/180 for each of the angles are included.

Figure 7 shows the path followed for the hexacopter, it is smooth with a fast response to disturbances, not only in altitude but also in the angles. The proposed control fulfills with flight and landing requirements set at the beginning of this work.
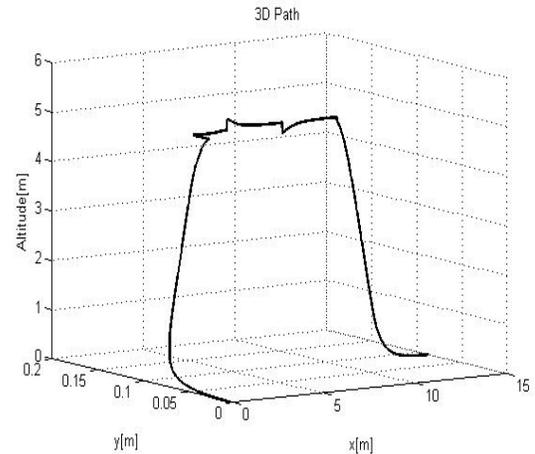
Figure 7. 3D path for altitude of the hexacopter with modeling errors and disturbances only for SMC

4.2.3 Effect of noise.

To test the effect of noise on the SMC, a Gaussian noise is added (Figure 8). The test shows that the SMC is sensitive to the presence of noise as it is expected from controller equation. The inclusion of noise increases the settling time.
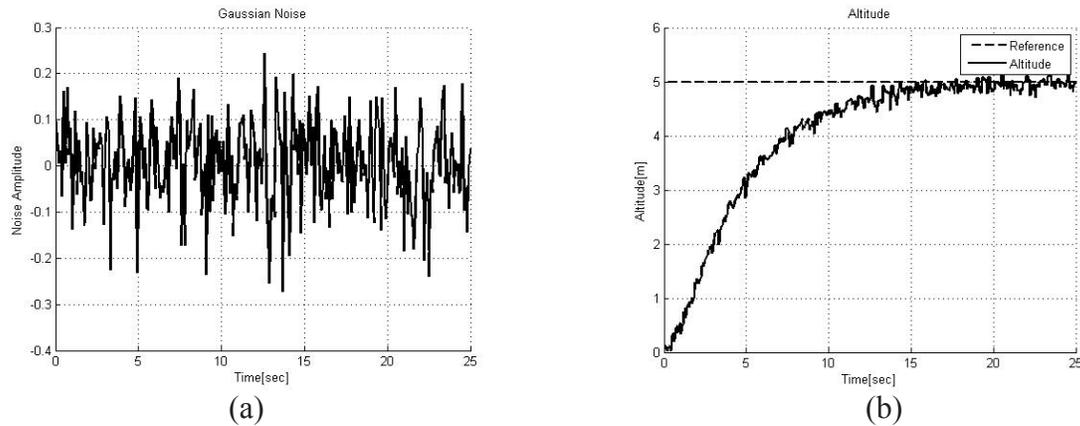
Figure 8. (a) Gaussian noise (b) Altitude response with noise

5. Conclusions

The present work has shown a robust control scheme based on Sliding Mode Control. The control scheme used is based on a classic PD control scheme used as the sliding surface.

The example presented indicate that the SMC performance is stable and quite satisfactory in spite of nonlinearities over a wide range of operating conditions.

The simulation results revealed that it is possible to implement the sliding mode control algorithm for hexacopter; the SMC presented a good performance for tracking and also robustness to modeling errors.

The proposed SMC was tested under noisy input signals, a common characteristics of real industrial processes. The SMC showed be tolerant to continuous noisy input signals, keeping a stable response behavior.

The tuning parameters for the SMC were found by trial and error, thence it is recommended to develop a tuning equations set to simplify the implementation.

Acknowledgments

References

[1]   HaiYang C., YongCan C., YangQuan C.: "Autopilots for Small Unmanned Aerial Vehicles: A Survey". International Journal of Control, Automation, and Systems. Vol. 8, No 1 (2010) 36-44.

[2]   Herrera M., Chamorro W., Gómez A., Camacho O.: "Sliding Mode Control: An approach to Control a Quadrotor". Asia-Pacific Conference on Computer Aided System Engineering (APCASE 2015). (2015).

[3]   Utkin, V. I., "Variable Structure Systems with Sliding Modes", Transactions of IEEE on Automatic Control, AC – No 22, (1977) 212 – 222.

[4]   Slotine, J.J., Li W.: "Applied Nonlinear Control", Prentice-Hall, New Jersey, (1991).

[5]   Zinober, A. S. I.: "Variable Structure and Liapunov Control", Spring – Verlag, London, (1994).

[6]   Camacho, O., Smith, C. and Chacón, E. Toward an Implementation of Sliding Mode Control to Chemical Processes. IEEE International Symposium on Industrial Electronics, (1997) 1101–1105.

[7]   Mohd A., Husain A., Kumeresan A.: "Robust Chattering Free Backstepping Sliding Mode Control Strategy for Autonomous Quadrotor Helicopter". International Journal of Mechanical & Mechatronics Engineering, Vol. 14, No. 03 (2014).

[8]   M'hammed G. and Hicham M.: "A High Gain Observer and Sliding Mode controller for an Autonomous Quadrotor Helicopter". International Journal of Intelligent Control and Systems, Vol.14, No. 3 (2009), 204-212.

[9]   Nader J., Mohammad R., Ali K.: "Robust Second Order Sliding Mode Control for a Quadrotor Considering Motor Dynamics". International Journal of Control Theory and Computer Modeling, Vol. 4, No.1/2 (2014).

[10]   Benallegue Y., Mokhtari A., Fridman L.: "High-order sliding-mode observer for a Quadrotor UAV". International Journal of Robust and Nonlinear Control, (2008) 427–440.

[11]   Arellano C., Luque L., Castillo B., Loukianov A.: "Backstepping control with Sliding Mode Estimation for a Hexacopter". 10th International Conference on Electrical Engineering, Computing Science and Automatic Control (CCE). (2013) 2-3.