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SETTLING VELOCITY MODELS APPLIED TO BALLASTED FLOCS – A REVIEW

MODELOS DE LA VELOCIDAD DE SEDIMENTACIÓN APLICADOS A FLÓCULOS LASTRADOS – UNA REVISIÓN

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ABSTRACT

The present study was conducted to evaluate the applicability of existing settling velocity models to ballasted flocs. An extensive literature review of the more common equations that represent settling velocity of flocs indicated that little work has been done about modeling the settling velocity of ballasted flocs. However, a general equation was found to be acceptable for this purpose, but the authors suggest the development of a new model that represent more accurately the settling velocity of ballasted flocs.

Key words: Settling velocity, flocculation, density, particle size, drag, permeability, fractals.

RESUMEN

El presente estudio fue realizado para evaluar la aplicabilidad de los modelos existentes de la velocidad de sedimentación para flóculos lastrados. Una extensa revisión en la literatura de las ecuaciones más comunes que representan la velocidad de sedimentación en flóculos indicó que se ha realizado poco trabajo en desarrollar modelos para la velocidad de sedimentación en flóculos lastrados. Sin embargo, se encontró una ecuación general que pudiera ser aceptable para este propósito, no obstante los autores sugieren el desarrollo de un nuevo modelo que represente con mayor precisión la velocidad de sedimentación de los flóculos lastrados.

Palabras clave: Velocidad de sedimentación, floculación, densidad, tamaño de partícula, arrastre, permeabilidad, fractales.

INTRODUCTION

The settling process consists of particles subjected to gravity and hydrodynamic forces falling through a liquid. Differential settling or differential momentum occurs when coagulated particles are vertically aligned and the difference between their settling velocities makes them collide and agglomerate. Differential settling contributes to enhanced flocculation and is mainly due to differences in size and density among particles. The addition of a ballasting agent significantly increases the particle density and thereby increases differential settling during the flocculation process. Differential settling is then very important when particle densities increase as they do in ballasted flocculation reactions. There is a lack of literature about the applicability of settling velocity models for ballasted flocs. Consequently, the purpose of the present work was to review existing settling velocity models found in the literature and evaluate their possible application for modeling the settling velocity of ballasted flocs. The resulting information is expected to be useful in modeling the settling velocity of ballasted flocs as used for water and wastewater treatment.

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GENERAL CONCEPTS ABOUT SETTLING VELOCITY

Sedimentation is often used for particle-fluid separation in water and wastewater treatment. In general, the design and operation of sedimentation processes requires an understanding of physical properties of these particles or flocs, such as floc density and size. Unhindered or free settling (Type 1) considers only the velocity of discrete particles settling alone under gravity forces. Under these conditions, the terminal settling velocity of the particle relative to fluid can be found using the following equation:

$$v_S = \left[\frac{2g(\rho_S - \rho)V_P}{C_D \, \rho A_P} \right]^{0.5} \qquad (1)$$

where v_S = terminal settling velocity (m/s); g = gravitational acceleration (9.81 m/s^2); ρ_S = particle density (kg/m^3); ρ = water density (kg/m^3), which is related to temperature; V_P = particle volume (m^3); A_P = particle projected area (m^2) and C_D = drag coefficient

(-). Therefore, Eq. 1 would be suitable for determining ρ_S if the other parameters were known. For this reason, settling velocity models have been developed by using free-settling tests, which simulate Type 1 settling (e.g. Tambo and Watanabe 1979, Li and Ganczarczyk 1987, Lee *et al*. 1996, Jiang and Logan 1991, Gorczyca and Ganczarczyk 1996, Johnson *et al*. 1996, Adachi and Tanaka 1997, Wu and Lee 1998).

For spherical particles, the ratio between the volume and the cross-sectional area of the particle perpendicular to the direction of flow can be calculated by:

$$\frac{V_P}{A_P} = \frac{2d}{3} \quad (2)$$

where d = particle diameter (m).

Therefore, introducing Eq. 2 into Eq. 1 yields a general equation for the terminal settling velocity of spherical particles, commonly known as Newton's law, in the form of:

$$v_S = \left[\frac{4g(\rho_S - \rho)d}{3C_D\rho}\right]^{0.5} \quad (3)$$

Floc Diameter

For practical purposes, floc size and floc diameter have been used as interchangeable terms. In general, values of v_S and d are determined experimentally from free-settling tests. Values of d can also be obtained using the concept of equivalent particle diameter or projected area diameter, d_a (m), which is the diameter of a circle having the same area as the particle when viewed from above and lying in its most stable position (Coulson and Richardson 1968, Allen 1975, Johnson *et al*. 1996). In this case,

$$d_a = \left(\frac{4A_P}{\pi}\right)^{0.5} \quad (4)$$

In which A_P is usually measured by microscopic observation.

According to Allen (1975), the settling velocity of a non-spherical particle determined by Eq. 3 may be evaluated as a function of the volume diameter, d_v (m), which is defined as "the diameter of a sphere having the same volume as the particle." Therefore, the value of d_v is used instead of d in Eq. 3. Using this definition, the volume of the particle is determined by:

$$V_P = \alpha_{v,a} d_a^3 = \frac{\pi}{6} d_v^3 \quad (5)$$

where $\alpha_{v,a}$ = volume-shape coefficient (-). For spherical particles,$\alpha_{v,a}$ = π/6.

Drag Coefficient

The drag coefficient, C_D, is a dimensionless number that depends on the distribution of the flow around the particle, the settling orientation, the Reynolds number, the shape of the particle and the permeability of the particle. The influence of the shape and permeability on C_D is discussed later in this article. The general equation for the Reynolds number, Re (-), in the range of $0 \leq \text{Re} \leq 10^5$ is defined as follows (Reynolds 1883):

$$Re = \frac{\rho v_S d}{\mu} \quad (6)$$

In this equation, d = particle diameter that corresponds to a characteristic length (m) and μ = absolute water viscosity (Pa-s or kg/m-s).

Gregory *et al*. (1999) presented a summary of conventional C_D values as a function of Re found in previous research by other investigators. These values are shown in Table 1.

Table 1. Variation of drag coefficient for impermeable spherical particles as a function of different Reynolds numbers for Type 1 settling[a]

Reynolds number, Re	Type of flow	Drag coefficient, C_D
$10^{-4} \leq \text{Re} < 1$	Laminar	$C_D = \frac{24}{\text{Re}}$ (7)
$1 \leq \text{Re} < 500\text{-}1000$	Transitional	$C_D = \frac{24}{\text{Re}} + \frac{3}{\text{Re}^{0.5}} + 0.34$ (8)
$500\text{-}1000 \leq \text{Re} < 2 \times 10^5$	Turbulent	~ 0.44
Re > 2 x 105	Turbulent	0.10

[a]Adapted from Gregory *et al*. (1999).

For laminar flow conditions (Re < 1), introducing Eq. 7 into Eq. 3 yields Stokes' law:

$$v_S = \frac{g(\rho_S - \rho)d^2}{18\mu} \quad (9)$$

Floc Shape

Flocs generated in water and wastewater treatment typically have irregular (fractal) geometries and different densities and sizes. Because of the complexity of analytical procedures for determining these parameters, sedimentation theories assume ideal conditions. For example, particles are considered spheres or shapes equivalent to spheres. A spherical particle presents the same shape to the oncoming flow independent of its orientation. Conversely, for a non-spherical particle this situation is different and its orientation as well as its shape will change according the flow condition: laminar, transitional, or turbulent (Coulson and Richardson 1968). Because the shape of the particle influences C_D (Brown *et al*. 1950, Allen 1975), some researches have included a shape factor, φ, in settling velocity models (e.g. Tambo and Watanabe 1979, Gorczyca and Ganczarczyk 1996, Johnson *et al*. 1996, Young and Edwards 2003) as follows:

$$v_S = \left[\frac{4g(\rho_S - \rho)d}{3C_D\varphi\rho}\right]^{0.5} \tag{10}$$

The shape factor is a dimensionless value that estimates how the projected area of a particle varies from a circle. Therefore, for spherical particles, φ is equal to unity. Common φ values are presented in Table 2.

Table 2 - Typical shape factor for different particles[a]

Particle	Shape factor, φ
Spheres	1
Sand	2
Coal	2.25
Gypsum	4
Graphite flakes	22

[a]Adapted from Degrémont (1991)

Other researchers have used different ways for calculating shape factors (ξ_1 and ξ_2) by using Eq. 11 (Swamee and Ojha 1991) and Eq. 12 (Námer and Ganczarczyk 1993, Gorczyca and Ganczarczyk 1996) respectively.

$$\xi_1 = \frac{l_3}{\sqrt{l_1 l_2}} \tag{11}$$

$$\xi_2 = \frac{4\pi A_P}{P_P^2} \tag{12}$$

where l_1, l_2, and l_3 are the length of the three principal axes of the particle in decreasing order of magnitude (m) and P_P = perimeter of the projected area of the particle (m).

Floc shape also has been related to floc sphericity, Ψ, which is a dimensionless value defined as the ratio between the surface area of a sphere having the same volume as the particle to the surface area of the particle (Brown *et al*. 1950, Allen 1975). The value of Ψ is equal to unity for spherical particles and less than unity for other shapes.

Another way to describe floc shape is by fractal geometry. The fractal dimension of a floc is a numerical representation of its highly irregular geometric shape. This irregular shape is attributed to the distribution of primary particles comprising the floc (Li and Ganczarczyk 1989). The influence of this fractal dimension in determining other floc characteristics such as density and permeability is discussed in the following sections.

Floc Density

Several researchers have found that floc density decreases as floc size increases, primarily, because the porosity and permeability of such aggregates increases as the floc size increases (Tambo and Watanabe 1979, Li and Ganczarczyk 1987, 1989, Zahid and Ganczarczyk 1990, Andreadakis 1993, Lee *et al*. 1996, Johnson *et al*. 1996). Li and Ganczarczyk (1989) have attributed these size-density relationships to the fractal dimension of flocs.

Tambo and Watanabe (1979) established a method for a quantitative evaluation of floc density, ρ_S (Eq. 3), using experimental procedures and a model for clay-aluminum flocs. The authors proposed the concept of floc effective density (buoyant density of floc), ρ_e (kg/m^3), to simplify the analysis of the nature of floc density as follows:

$$\rho_e = \rho_S - \rho \tag{13}$$

The ρ_e value was determined from a settling velocity equation derived from Eq. 3 (see Table 3). From experimental results, one of the conclusions was that ρ_e is a function of the log of d, according to:

$$\rho_e = \frac{c}{(d/1)^{K_P}} \tag{14}$$

This relationship was designated as the floc density function in which c = constant that represents the effective density of 0.1 m diameter floc (kg/m^3); $(d/1)$ = dimensionless floc diameter (m/m) and K_P = slope of the line that describes the floc density function (-). Others researchers have reported a similar relationship (Lagvankar and Gemmell 1968, Li and Ganczarczyk 1989, Zahid and Ganczarczyk 1990).

Tambo and Watanabe (1979) also verified the floc density function, which was obtained by substituting the number of primary particles contained in a floc and d into a mass balance equation and the results were compared to experimental data.

Using an analytical method known as interference microscopy, for obtaining the bulk density of activated sludge flocs, Andreadakis (1993) also found the same relationship between floc size and density as expressed in Eq. 14.

Floc Permeability

Some studies have demonstrated that increased floc permeability reduces drag forces and therefore, increases settling velocities (Neale *et al*. 1973, Matsumoto and Suganuma 1977, Masliyah and Polikar 1980, Li and Ganczarczyk 1988, Johnson *et al*. 1996). For this reason, the application of Stokes' law, which does not take into account the permeability has been questioned for determining ρ_S (Wu and Lee 1998).

The effect of permeability on v_S of a porous sphere under laminar flow conditions (Re < 1) can be evaluated using a dimensionless correction factor Ω (Neale *et al*. 1973, Matsumoto and Suganuma 1977, Lee *et al*. 1996, Wu and Lee 1998). This factor represents the hydrodynamic resistance of flow through a permeable sphere and, is defined as the ratio between the resistance experienced by a permeable sphere to that experienced by an impermeable sphere, both having the same radius. Thus for impermeable spheres, Ω is equal to unity.

The formula used to determine Ω, originally developed by Brinkman and corrected by Debye, is presented in Neale *et al*. (1973) as follows:

$$\Omega = \frac{2\beta^2\left[1-(\tanh\beta)/\beta\right]}{2\beta^2+3\left[1-(\tanh\beta)/\beta\right]} \quad (15)$$

where, β, the dimensionless particle diameter, is defined as follows:

$$\beta = \frac{d}{2\sqrt{k}} \quad (16)$$

in which k is the floc permeability (m^2).

A general expression for floc permeability is represented by (Lee *et al.* 1996):

$$k = d_P^2\, f(e) \quad (17)$$

in which d_P is the characteristic length of the primary particles forming the floc (m) and $f(e)$ is a function of the floc porosity.

A good discussion of different permeability models appears in Lee *et al.* (1996). Although Eq. 15 has been used in subsequent studies, the validity of this model is still in question for non-spherical permeable particles and non-laminar flow conditions (Re ≥ 1). Because the correlation between C_D and Ω for Re ≥ 1 is deficient in the literature, Lee *et al*. (1996) suggested the need for more research with respect to this correlation.

Because of the influence of fractal dimension on k, investigators have also included fractal dimension theories for developing settling velocity models (Li and Ganczarczyk 1988, 1989, 1992, Johnson *et al*. 1996, Lee *et al*. 1996, Wu and Lee 1998).

Johnson *et al*. (1996) observed a reduction in drag of permeable fractal aggregates compared to that of impermeable spheres or permeable spheres. As a result, they found that measured values of v_S for the fractal aggregates were higher than those predicted by using Stokes' law for impermeable spheres or by using another model for permeable spheres of identical mass, cross-sectional area, and primary particle density. This finding was not in agreement with other studies and this reduction of drag was not the same as that described by a shape factor. Johnson *et al*. (1996) suggested that permeability relationships used by other investigators (e.g. Masliyah and Polikar 1980) for predicting v_S of permeable aggregates would be incorrect if applied to fractal aggregates because such permeability models have been developed for aggregates composed of particles distributed homogeneously throughout the aggregate. Johnson *et al*. (1996) attributed the differences in v_S between fractal aggregates and permeable aggregates to a heterogeneous distribution of primary particles in fractal aggregates. This heterogeneous distribution was thought to be the result of differences in density between small packed clusters forming larger aggregates and these aggregates. Consequently, in a fractal aggregate, the permeability of the macropores between such clusters is probably greater than the permeability inside the clusters.

Based on the hypothesis proposed by Lee *et al*. (1996) with respect to a permeable floc moving at Re greater than unity (non-laminar flow), Wu and Lee (1998) evaluated the drag force exerted on individual floc under quiescent conditions for Re numbers between 0.1 and 40. The numerical solution for the model, which was developed to evaluate the drag force, revealed that at β values greater than approximately 50 and Ω

= 1 (nonporous sphere), Re had almost no effect on Ω. Whereas, for β values lower than 10 (porous sphere), there was a considerable reduction in Ω with increasing Re and therefore a decrease in drag force. Because of the high permeability of sludge flocs, they suggested that a Stokes' law-like correlation was applicable for estimating fractal dimension of the flocs based on floc size-density relationships. However, floc density could not be accurately estimated from free-settling tests because of the lack of floc permeability data.

SETTLING VELOCITY MODELS

Because of the influence of φ and k on C_D, and consequently on v_S, other expressions besides Stokes' law (Eq. 9) or Eq. 10 have been suggested (Table 3). A comparison of these settling velocity models shows inconsistency among the several equations for determining v_S. Consequently, there is not an accepted universal expression that describes the settling velocity behavior of fractal aggregates. One of the reasons for this discrepancy is the utilization of different theoretical assumptions for each proposed model. In addition, those equations that include fractal dimensions (e.g. Li and Ganczarczyk 1987, Jiang and Logan 1991, Gorczyca and Ganczarczyk 1996, Adachi and Tanaka 1997, Wu and Lee 1998) have v_S as a function of the equivalent diameter or the longest dimension of the flocs to an exponent (fractal dimension). According to Eq. 10, the constants of the fractal dimension equations would include variables such as ρ_S which depends on floc size, and C_D, which depends on φ and k. Consequently, the constant could not possibly be the same in each equation or be assumed as a constant coefficient for defining a general equation. On the other hand, other researchers have assumed a constant value for C_D, which could be wrong because of the different Re values and the influence of the other variables previously mentioned and this statement could be supported by Lee *et al*. (1996) who suggested,

> "The drag coefficient expression employed might be the most influencing factor affecting the size-density relationship."

Lawler and Wilkes (1984) explained that settling velocity of flocculated particles can be determined from the solution of Eq. 9, when Re < 1 (laminar flow), and from Eqs. 3, 6, and 8 by a trial-and-error technique, when Re ≥ 1 (non-laminar flow).

Table 3. Settling velocity models

Particle	Drag coefficient	Reynolds Number	Settling velocity (LT^{-1})	Reference
Clay-aluminum	$C_D = \frac{45}{Re}$	< 10^6	$v_S = \frac{g(\rho_S - \rho)d^2}{34\mu}$	Tambo and Watanabe (1979)
Activated sludge	NS	NS	$v_S = 0.35 + 1.77$ d $v_S = 0.33 + 1.28$ L $v_S = 1.47L^{0.55}$	Li and Ganczarczyk (1987)
Activated sludge	$C_D = \frac{K}{Re}$	< 1	$v_S = \left(\frac{4g(\rho_S - \rho)d}{3C_D\Omega\rho}\right)^{0.5}$	Lee *et al*. (1996)
	$C_D = 0.28\left(1 + \frac{9.06}{\sqrt{Re}}\right)^2$ [a]	1-1000 [b]	$v_S \propto d^{0.7\text{-}0.8}$	
	$C_D = \frac{24\Omega}{Re}\left[1 + 0.1315 Re^{(0.82-0.05w)}\right]$	0.1-7 [c]		
	$C_D = \frac{24\Omega}{Re}\left[1 + 0.0853 Re^{(1.093-0.105w)}\right]$	7-120 [c]		
Fractal aggregates	$C_D = \frac{a}{Re^b}$ [d]	0.1-10	$v_S \sim L^{(D3-D2+b)/(2-b)}$	Jiang and Logan (1991)
Fractal aggregates	$C_D = \frac{a}{Re^b}$ [d]	0.1-10	$v_S = \frac{g(\rho_S - \rho)NV_0}{3\pi\upsilon\rho d}$ [b] $v_{Sperm} = \frac{v_S}{\Omega}$ [c]	Johnson *et al*. (1996)
Aluminum-kaolinite floc	$C_D \sim \alpha$ [e]	NS	$v_S = 4.946\, D_v^{1.379}$ $v_S = 0.768\, D_m^{1.149}$ $v_S = 0.969\, D_e^{1.150}$	Adachi and Tanaka (1997)
Activated sludge	$C_D = \frac{a(\beta)}{\Omega Re}$	0.1-40	$v_S = 1.17\, d^{0.99}$	Wu and Lee (1998)

NS = Not specified. [a]This expression is the same as Eq. 8. [b]Impermeable spheres. [c]Permeable spheres. [d] For Re≤0.1, a = 24 and b = 1; for 0.1<Re<10, a = 29.03 and b = 0.871; for 10<Re<100, a = 14.5 and b = 0.547. [e]Proportional constant for the drag that includes shape and permeability.

Based on the reasons previously explained in this section, one could assume that Eq. 10 is still valid as a general expression for determining ballasted floc characteristics in settling velocity tests. However, this equation should also include Ω as a correction factor for $C_{D.}$

SETTLING VELOCITY IN BALLASTED FLOCCULATION

Young and Edwards (2003) conducted a series of settling-velocity tests to determine the effect of ballasting agent on the settling velocity of the resulting floc. For these tests, ρ_S was assumed to vary between 1.2 g/cm^3 and 2.0 g/cm^3 and φ values were held equal to 2.0. Results showed that particles having diameters between 0.5 mm and 7.0 mm achieved settling velocities ranging from about 100 m/hr to as high as 380 m/hr respectively. Microsand particles with diameters up to 0.4 mm had measured settling velocities values in agreement with those calculated from the model represented by Eq. 10 when using a ρ_S of 2.65 g/cm^3 and a φ of 2.0.

Young and Edwards (2003) explained that other combinations of ρ_S and φ could produce the same calculated curves. However, if φ values were increased, it would require proportionally higher ρ_S values, which would not be realistic. Finely ground granular activated carbon (GAC) also was used as a ballasting agent in similar tests. High settling velocities were also achieved with no significant differences between doses (2.0 and 5.0 g GAC/L).

An increase in settling velocities of ballasted activated sludge flocs compared to conventional flocs, also was observed in other studies conducted by Chang *et al*. (1998), Piirtola *et al*. (1999a,b), and Ramsay (1995).

CONCLUSIONS

The literature review presented above, lead to the conclusion that little work has been done about evaluating the applicability of settling velocity equations for modeling the settling velocity of ballasted flocs. However, this velocity can be expressed by Eq. 10 and applied to model ballasted floc settling velocity as suggested by Young and Edwards (2003). But, the need for assessing this problem more deeply suggest the development of a test program designed with the purpose of verifying the application of Eq. 10 for ballasted flocs and the relationship between ballasted floc size, density, shape, and permeability.

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